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Syllabus for the M.Phil./Ph.D. Entrance Test

Probability Theory

Classes of sets, field, sigma field, minimal sigma field, Borel field, sequence of sets, limits of a sequence of sets, measure, probability measure, Integration with respect to measure.

Basic, Markov's, Holder's, Minkowski's and Jensen's inequalities.

Random variables, convergence of a sequence of random variables-convergence in probability, almost surely, in the rth mean and in distribution, their relationship, Helly-Bray theorem, monotone convergence theorem, Fatou's lemma, dominated convergence theorem, three-series criterion.

Characteristic function, uniqueness theorem, continuity theorem, inversion formula.

Laws of large numbers, Chebyshev's and Khinchine's WLLN, necessary and sufficient condition for the WLLN, Kolmogorov and Hajek-Renyi inequalities, strong law of large numbers and Kolmogorov's theorem.

Central limit theorem, Lindeberg and Levy and Liapunov forms of CLT. Statement of Lindeberg and Feller's CLT and examples.

Definition and examples of Markov dependence, exchangeable sequences, m-dependent sequences, stationary sequences.

Statistical Methodology

Brief review of basic distribution theory. Symmetric Distributions, truncated and compound distributions, mixture of distributions, Power series distribution, exponential family of distributions, Characterization of distributions (Geometric, negative exponential, normal, gamma), non-central chi-square, t and F distributions and their properties, Concept of censoring. Approximating distributions, Delta method and its applications. Approximating distributions of sample moments, limiting moment generating function, Poisson approximation to negative binomial distribution. Order statistics - their distributions and properties. Joint and marginal distributions of order statistics. Extreme values and their asymptotic distributions (statement only) with applications. Tolerance intervals, coverage of $(X_{(r)},X_{(s)})$. General theory of regression, fitting of polynomial regression by orthogonal methods, multiple regression, examination of regression equation. Robust procedures, robustness of sample mean, sample standard deviation, chi-square test and Student's t-test. Sample size determination for testing and estimation procedures (complete and censored data) for normal, exponential, Weibull and gamma distributions.

Survey Sampling

Basic ideas and distinctive features of sampling; Probability sampling designs, sampling schemes, inclusion probabilities and estimation; Fixed (Design-based) and Superpopulation (model-based) approaches; Review of important results in simple and stratified random sampling; Sampling with varying probabilities (unequal probability sampling) with or without replacement –pps, πps and non-πps sampling procedures and estimation based on them; Non-negative variance estimation; Two-way stratification, post-stratification, controlled sampling; Estimation based on auxiliary data (involving one or more auxiliary variables) under design-based and model-based approaches; Double (two-phase) sampling with special reference to the selection with unequal probabilities in at least one of the phases; systematic sampling and its application to structured populations; Cluster sampling (with varying sizes of clusters); Two-stage sampling (with varying sizes of first-stage units).

Non-sampling errors with special reference to non-response, Warner's and Simmons' randomized response techniques for one qualitative sensitive characteristic.

Stochastic Processes

Poisson process, Brownian motion process, Thermal noise, Markov-shot noise, Two-valued processes. Model for system reliability.

Mean value function and covariance kernel of the Wiener and Poisson processes. Increment process of a Poisson process, Stationary and evolutionary processes.

Compound distributions, Total progeny in branching processes.

Recurrent events, Delayed recurrent events, Renewal processes. Distribution and Asymptotic Distribution of Renewal Processes. Stopping time. Wald's equation. Elementary Renewal Theorem. Delayed and Equilibrium Renewal Processes. Application to the theory of success runs. More general patterns for recurrent events.

One-dimensional, two-dimensional and three-dimensional random walks. Duality in random walk. Gambler's ruin problem.

Classification of Markov chains. Higher transition probabilities in Markov classification of states and chains. Limit theorems. Irreducible ergodic chain.

Martingales, Boob- Decomposition, Martingale convergence theorems, Optional stopping theorem.

Statistical Inference

Minimal sufficiency and ancillarity, Exponential families and Pitman families, Invariance property of Sufficiency under one-one transformations of sample and parameter spaces. Fisher Information for one and several parameters models. Lower bounds to variance of estimators, necessary and sufficient conditions for MVUE.

Neyman-Pearson fundamental lemma and its applications, UMP tests for simple null hypothesis against one-sided alternatives and for one-sided null against one-sided alternatives in one parameter exponential family. Extension of these results to Pitman family when only upper or lower end depends on the parameters and to distributions with MLR property, Non-existence of UMP tests for simple null against two-sided alternatives in one parameter exponential family. Families of distributions with monotone likelihood ratio and UMP tests.

Interval estimation, confidence level, construction of shortest expected length confidence interval, Uniformly most accurate one-sided confidence Interval and its relation to UMP tests for one-sided null against one-sided alternative hypotheses.

Consistency and asymptotic relative efficiency of estimators. Consistent asymptotic normal (CAN) estimator. Method of maximum likelihood, CAN estimator for one parameter Cramer family, Cramer-Huzurbazar theorem. Solutions of likelihood equations, method of scoring. Fisher lower

bound to asymptotic variance. MLE in Pitman family and double exponential distribution, MLE in censored and truncated distributions.

Similar tests, Neyman structure, UMPU tests for composite hypotheses, Invariance tests and UMP invariant tests, Likelihood ratio test, Asymptotic distribution of LRT statistic, Consistency of large sample test, Asymptotic power of large sample test.

Sequential tests-SPRT and its properties, Wald's fundamental identity, OC and ASN functions. Sequential estimation.

Non- parametric methods-estimation and confidence interval, U-statistics and their asymptotic properties, UMVU estimator, non parametric tests-single sample location, location-cum-symmetry, randomness and goodness of fit problems; Rank order statistics, Linear rank statistics, Asymptotic relative efficiency.

Design of Experiments

Review of linear estimation and basic designs. ANOVA: Fixed effect models (2-way classification with unequal and proportional number of observations per cell), Random and Mixed effect models (2-way classification with m (>1) observations per cell).

Incomplete Block Designs. Concepts of Connectedness, Orthogonality and Balance. Intrablock analysis of General Incomplete Block design. B.I.B designs with and without recovery of interblock information.

Elimination of heterogeneity in two directions.

Symmetrical factorial experiments (s^m, where s is a prime or a prime power), Confounding in s^m factorial experiments, s^{k-p} fractional factorial where s is a prime or a prime power. Split-plot experiments.

Finite fields. Finite Geometries- Projective geometry and Euclidean geometry. Construction of complete set of mutually orthogonal Latin squares.

Construction of B.I.B.D. using finite Abelian groups, MOLS, finite geometry and method of differences.

Multivariate Analysis

Multivariate normal distribution, its properties and characterization. Random sampling from a multivariate normal distribution. Maximum likelihood estimators of parameters. Distribution of sample mean vector. Inference concerning the mean vector when the covariance matrix is known. Matrix normal distribution. Multivariate central limit theorem.

Wishart matrix — its distribution and properties. Distribution of sample generalized variance.

Hotelling's T^2 statistic — its distribution and properties. Applications in tests on mean vector for one and more multivariate normal populations and also on symmetry of organs. Mahalanobis' D^2.

Likelihood ratio test criteria for testing (1) independence of sets of variables, (2) equality of covariance matrices, (3) identity of several multivariate normal populations, (4) equality of a covariance matrix to a given matrix, (5) equality of a mean vector and a covariance matrix to a given vector and a given matrix.

Distribution of the matrix of sample regression coefficients and the matrix of residual sum of squares and cross products. Rao's U-statistic, its distribution and applications.

Classification and discrimination procedures for discrimination between two multivariate normal populations — sample discriminant function, tests associated with discriminant functions, classification into more than two multivariate normal populations.

Principal components, canonical variables and canonical correlations. Elements of factor analysis and cluster analysis.

Multivariate linear regression model – estimation of parameters and their properties. Multivariate analysis of variance [MANOVA] of one-way classified data. Wilk's lambda criterion.

Econometrics and Time Series Analysis

Time series as discrete parameter stochastic process. Auto covariance and auto correlation functions and their properties.

Detailed study of the stationary processes : Moving average (MA), Auto regressive (AR), ARMA and ARIMA models. Box-Jenkins models. Discussion (without proof) of estimation of mean, auto covariance and auto correlation functions under large sample theory. Choice of AR and MA periods. Estimation of ARIMA model parameters. Smoothing spectral analysis of weakly stationary process. Periodo gram and correlogram analysis. Filter and transfer functions. Problems associated with estimation of spectral densities.

Forecasting: Exponential and adaptive Smoothing methods

Econometrics: Review of G.L.M. and generalized least squares estimation, GLM with stochastic regressors. Instrumental variables, estimation, consistency property, asymptotic variance of instrumental variable estimators.

Bayesian analysis of G.L.M. with informative and non informative prior distributions. Bayes estimation and testing of hypotheses of the regression coefficients.

Distributed lag models: Finite polynomial lags, determination of the degree of polynomial. Infinite distributed lags, adaptive expectations and partial adjustment models, determination of lag length. Methods of estimation.

Simultaneous equations models: Identification problem. Restrictions on structural parameters-rank and order conditions. Restrictions on variances and covariances. Estimation in simultaneous equations models. Recursive systems, 2SLS estimators, Limited information estimators, k-class estimators, Instrumental variable method of estimation. 3-SLS estimation.

Demography, Statistical Quality Control and Reliability

Demography: Measures of mortality, description of life table, construction of complete and abridged life tables, maximum likelihood, MVU and CAN estimators of life table parameters.

Measures of fertility, models for population growth, intrinsic growth rate, stable population analysis, population projection by component method and using Leslie matrix.

Quality control and Sampling Inspection: Basic concepts of process monitoring and control, General theory and review of control charts, O.C and ARL of control charts, CUSUM charts using V-mask and decision intervals, economic design of x- bar chart.

Review of sampling inspection techniques, single, double, multiple and sequential sampling plans and their properties, methods for estimating (n, c) using large sample and Bayesian techniques, curtailed and semi-curtailed sampling plans, Dodge's continuous sampling inspection plans for inspection by variables for one-sided and two-sided specifications.

Reliability: Reliability concepts and measures, components and systems, reliability function, hazard rate, common life distributions viz. exponential, gamma, Weibull, lognormal, Rayleigh, piece-wise exponential etc., Reliability and expected survivability of series, parallel, mixed, maintained and non-maintained systems with and without redundancy, preventive maintenance policy, preliminary concepts of coherent systems.